MATH 2850: SOLVING IVPs USING LAPLACE TRANSFORMS

THEOREM: $\mathcal{L}\{y'(t)\} = sY(s) - y(0)$

PROOF:

STRATEGY: Given an IVP:

1. Use Laplace Transforms to turn the IVP in the t domain into an algebraic equation in the s domain.
2. Solve the algebraic equation in the s domain.
3. Use the Inverse Laplace Transform to bring the solution in the s domain back to the t domain.

EXAMPLE: Solve: $y' - 6y = 3t$, $y(0) = 2$.

Ans: $y = \dfrac{25}{12}\, e^{6t} - \dfrac{1}{2}\, t - \dfrac{1}{12}$

EXAMPLE: Use the fact that $\mathcal{L}\{y'(t)\} = sY(s) - y(0)$ to show $\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$.

EXAMPLE: Solve: $y'' - y' - 6y = 0$; $y(0) = 1$, $y'(0) = 0$.

Ans: $y = \dfrac{3}{5}\,e^{-2t} + \dfrac{2}{5}\,e^{3t}$

EXAMPLE: Solve: $y'' - 4y' + 4y = 5$; $y(0) = 0$, $y'(0) = 1$

Ans: $y = -\dfrac{5}{4}\,e^{2t} + \dfrac{7}{2}\,t\,e^{2t} + \dfrac{5}{4}$

EXAMPLE: Solve: $y'' - 4y' + 4y = 100\sin(4t) + 8t + 4$; $y(0) = 8$, $y'(0) = 9$

Ans: $y = e^{2t} + 17t\,e^{2t} - 3\sin(4t) + 4\cos(4t) + 2t + 3$

HOMEWORK: Section 8.3: Pg. 419: 1 - 33 every other odd.